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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share "work in progress" in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschafts-wissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

Abstract

A discrete time model of financial markets is considered. It is assumed that the stock price evolution is described by a homogeneous Markov chain. In the focus of attention is the expected value of the guaranteed profit of the investor that arises when the jumps of the stock price are bounded. The suggested diffusion approximation for the Markov chain allows establishing a convenient approximate formula for the studied characteristic.

Keywords

Ergodic and irreducible Markov chains, stationary distribution, local limit theorem, upper hedge, upper rational price

JEL Classification

G12, G11, G13

Comments

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Contents

1 Introduction

Consider the simplest financial market in which securities of two types are circulating. The price evolution of the securities of the first type is given by the equations

$$b_k = b_0 \rho^k, \ k = 0, 1, 2, \ldots,$$

where $b_0 > 0, \ \rho > 1$. The prices are registered at the equidistant moments of time $t_k = a + kh$. With no loss of generality we put $a = 0, \ h = 1$, i.e. $t_k = k$.

The price of the security of the second type at the moment k is represented as

$$s_k = s_0 \xi_1 \cdots \xi_k, \ k = 0, 1, 2, \ldots,$$

where the relative jumps ξ_k are random.

The securities of the first type are *riskless* having the interest rate $(\rho - 1) \cdot 100\%$. Let us call them conventionally *bonds.* It is clear that possessing the securities of the second type is concerned with a risk of their devaluation. We call them conditionally *stocks.*

Taken together in certain amounts β and γ the securities of both types constitute a so-called *portfolio (writer's investment portfolio)* whose worth at the time moment k is $\beta b_k + \gamma s_k$. *Playing* in the considered financial market consists of successive changing of the portfolio content at the moments $k = 1, 2, \ldots, n-1$. The successive pairs $(\beta_0, \gamma_0), \ (\beta_1, \gamma_1), \ldots, (\beta_{n-1}, \gamma_{n-1})$ constitute a so-called *strategy* of the game. Obviously, as a basis for choosing (β_k, γ_k) serves the evolution of the stock price up to this moment i. e. $s_0, s_1, \ldots, s_k$. In other words

$$\beta_k = \beta_k(s_0, s_1, \ldots, s_k), \ \gamma_k = \gamma_k(s_0, s_1, \ldots, s_k).$$

The player is called a *writer (seller, investor).*

A strategy is called *self-financing* if the changing of the portfolio content does not affect its value i.e.

$$\beta_k b_k + \gamma_k s_k = \beta_{k-1} b_k + \gamma_{k-1} s_k, \ k = 1, \ldots, n-1.$$

The final goal of the game is to meet the condition

$$x_n = \beta_{n-1} b_n + \gamma_{n-1} s_n \geq f(s_n) \tag{1.1}$$

where $f(s)$ is a so-called *pay-off function* of the simplest option of the *European* type having n as a *maturity date.*

Basic problems of the mathematical theory of options are option pricing and building a strategy leading to (1.1). For more about the mathematical and substantial aspects of the option pricing theory see, e.g., Shiryaev (1999).

Both problems are easily solved within the framework of the so-called *binary* model, that is, in the case where ξ_k take only two values d and u, $d < \rho < u$. In this case (see, e.g., Ch. VI in Shiryaev (1999))

$$x_0 = \rho^{-n} \sum_{k=0}^{n} C_n^k p_*^k (1 - p_*)^{n-k} f(s_0 u^k d^{n-k}) \tag{1.2}$$

where

$$p_* = \frac{\rho - d}{u - d}.$$

It is worth emphasizing that (1.2) does not assume any restrictions imposed on the measure that governs the evolution of the stock price $(\xi_1, \ldots, \xi_n)$. Furthermore, there exists the unique self-financing strategy

$$(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \{(\beta_0, \gamma_0),\ (\beta_1, \gamma_1), \ldots, (\beta_{n-1}, \gamma_{n-1})\}$$

leading to the equality

$$x_n = \beta_{n-1} b_n + \gamma_{n-1} s_n = f(s_n). \tag{1.3}$$

The strategy is defined by the formulae

$$\beta_k = \frac{u f_{k+1}(s_k d) - d f_{k+1}(s_k u)}{\rho b_k (u - d)} \tag{1.4}$$

and

$$\gamma_k = \frac{f_{k+1}(s_k u) - f_{k+1}(s_k d)}{s_k (u - d)} \tag{1.5}$$

where

$$f_k(s) = \rho^{-(n-k)} \sum_{j=0}^{n-k} C_{n-k}^j p_*^j (1 - p_*)^{n-k-j} f(s u^j d^{n-k-j}). \tag{1.6}$$

The successive values of the portfolio are

$$x_k = f_k(s_k),\ k = 0, 1, \ldots, n-1.$$

If ξ_k, $k = 1, 2, \ldots, n$, take more than two values then it is impossible to guarantee the desired relation (1.3) with probability 1. However, sometimes it is possible to guarantee (1.1). Let $f(s)$ be convex. Then from (1.6) it follows that all $f_k(s), k = 0, 1, \ldots, n-1$, also are convex. If, furthermore, if $\xi_k \in [d, u]$ then the minimal initial capital is evaluated by the same formula (1.2).

This fact was, first, proven in Tessitore and Zabczyk (1996) by the methods of control theory (see also Zabczyk (1996) and Motoczyński and Stettner (1998)). Later on in Shiryaev (1999) the rational price is derived as the solution of a extreme problem (see Theorem V.1c.1 ibidem).

Denote

$$\bar{x}_k = f_k(s_k),\ k = 0, \dots, n-1, \tag{1.7}$$

and let (β_k, γ_k) be defined as in (1.4) and (1.5).

Possessing after the $(k-1)-$th step the capital $\bar{x}_{k-1}$ distributed in portfolio in accordance with (1.4) and (1.5) at the next step k the investor gains the capital

$$x_k = \beta_{k-1} b_k + \gamma_{k-1} s_k = \frac{u - \xi_k}{u - d} f_k(s_{k-1} d) + \frac{\xi_k - d}{u - d} f_k(s_{k-1} u).$$

If $\xi_k \in [d, u],\ k = 1, \dots, n,$ and $f(s)$ is convex then

$$\delta_k = x_k - \bar{x}_k = f_k(s_{k-1} d)\frac{u - \xi_k}{u - d} + f_k(s_{k-1} u)\frac{\xi_k - d}{u - d} - f_k(s_{k-1}\xi_k) \geq 0. \tag{1.8}$$

If $f_k(s_{k-1}\xi)$ is strictly convex in $[d_k, u_k]$ then $\delta_k = 0$ if and only if $\xi_k = d$ or $\xi_k = u$. Otherwise $\delta_k > 0$. Thus, if ξ_k takes at least one value lying in (d, u) then a profit can arise. If the extreme values d and u belong to the support of the distribution of ξ_k then $\bar{x}_{k-1}$ is the minimal capital which allows such a profit. It implies that $\bar{x}_0$ is the minimal starting capital that allows the investor to meet his contract obligations with probability 1 provided he follows the strategy determined by (1.4) and (1.5). This strategy forms the so-called *upper hedge*. It determines the sequence $(\bar{x}_0, \bar{x}_1, \dots, \bar{x}_{n-1})$ of is the corresponding chain of *hedging capitals*. Here, $\bar{x}_0$ is called the *upper rational price.*

The investor may dispose of the so arisen profit in various ways. The simplest one is to withdraw from the game the superfluous quota δ_k which to the maturity date acquires the value $\delta_k \rho^{n-k}$. So, the self-financing condition is fulfilled only in the part which bans any capital inflows.

Having withdrawn unnecessary quota one should follow the "binary" optimal strategy determined by (1.4) and (1.5). As a result to the maturity date the investor accumulates a riskless profit

$$\Delta_n = \delta_1 \rho^{n-1} + \delta_2 \rho^{n-2} + \cdots + \delta_n.$$

It should be emphasized that the upper hedge admits an *arbitrage* opportunity in the sense that the investor always meet his obligations, i.e.

$$\mathrm{P}(x_n \geq f(s_n)) = 1,$$

and may have a riskless profit

$$\mathrm{P}(\Delta_n > 0) > 0.$$

In the present paper, as in A. Nagaev and S. Nagaev (2003) and S. Nagaev (2003), we study the distribution of Δ_n. The results established in the referred works are based on the assumption that the relative jumps of the stock price $\xi_1, \xi_2, \dots, \xi_n$ are i.i.d. random variables. It is natural to try to extend those results to models that assume some kind of dependence between those jumps. Here, we assume that they form a Markov chain. Intuitively, we expect that the methods worked out in the referred works allow analysis of such scheme provided the Markov chain is sufficiently regular.

The paper is organized as follows. In Section 2 we introduce a model of financial market in which the evolution of the risky security is described by a Markov chain with a finite number of states. Further, we state the basic results concerning the expected value of the riskless profit under selling the call and put options. The formulae for the "local" profit are established in Section 3. The basic statement is proven in Section 4. In Section 5 some auxiliary results are given. Concluding remarks are given in Section 6.

2 The model description and main result

Let $\omega_0, \omega_1, \omega_2, \dots$ be a Markov chain having the phase space $I = \{1, 2, \dots, M\}$. Assume that the chain is homogeneous and starts from i_0.. Denote by P the transition matrix, i.e.

$$P = ||P_{ik}||_{i,k=1}^M.$$

The measure on the trajectories $\omega_1, \omega_2, \dots,\ \omega_n$ is defined as follows

$$\mathrm{P}_{i_0}(\omega_1 = i_1,\ \omega_2 = i_2, \dots, \omega_{n-1} = i_{n-1},\ \omega_n = i_n) = p_{i_0,i_1} p_{i_1,i_2} \cdots p_{i_{n-1},i_n}.$$

We assume also that the chain is irreducible and ergodic or, in other words, the matrix P is *primitive*, i.e. there exists an integer $n_0 \geq 1$ such that all the entries $p_{ik}^{(n_0)},\ i, k = 1, \dots, M$, of P^{n_0} are positive. Denote by $\pi = (\pi_1, \dots, \pi_M)$ the stationary distribution of the considered chain.

Let g be a function defined on I, i.e. $g:\ I \to I\!R$. This function determines the sequence of random variables $\eta_k = g(\omega_k),\ k = 0, 1, 2, \dots$. Without loss of generality we assume that $g(1) \leq g(2) \leq \cdots \leq g(M)$ and

$$\sum_{i=1}^M g(i)\pi_i = 0.$$

Denote $y = -g(1),\ x = g(M)$. If g is not constant on I then $g(1) < 0 < g(M)$.

Consider the random process

$$z_n(t) = hkn^{-1} + n^{-1/2}(\eta_1 + \cdots + \eta_k),\ (k-1)n^{-1} \leq t < kn^{-1}$$

where h is a constant. Obviously, the process takes values in $D[0, 1]$.

Proposition 2.1 *If g is such*

$$\sigma^2 = \sigma_g^2 = \sum_{l=1}^{M} g^2(l)\pi_l + 2\sum_{n=1}^{\infty}\sum_{l,k=1}^{M} g(l)g(k)\pi_l p_{lk}^{(n)} > 0, \tag{2.9}$$

then the random process $z_n(t),\ 0 \le t < 1$, weakly converges to $z(t) = ht + \sigma w(t)$ wher $w(t)$ is the standard Wiener process.

The proof of this statement is the evident modification of those given in Ch. 2.3 of Sirazhdinov and Formanov (1979) (see also Billingsley (1956) and Friedman (1967). The following statement is the univariate version of Theorem 5.4 in Sirazhdinov and Formanov (1979).

Proposition 2.2 *Assume that the set $\{g(1), g(2), \dots, g(M)\}$ is not* lattice. *Then under the conditions of Proposition 2.1 the measures $\sigma\sqrt{2\pi n}\mathrm{P}_{i_0}(\eta_1 + \dots + \eta_n \in A)$ weakly converge as $n \to \infty$ to the Lebesgue measure.*

Assume that

$$\begin{cases} \xi_k = \xi_{k,n} & = \exp(hn^{-1} + \eta_k n^{-1/2}) \\ \\ \rho = \rho_n & = \exp(\alpha n^{-1}) \end{cases} \tag{2.10}$$

where h and $\alpha > 0$ are constant, while the random variables $\eta_1, \dots, \eta_n$, are defined on the successive states of the above Markov chain. So, $\xi_k = \xi_{k,n} \in [d_n, u_n],\ k = 1, 2, \dots, n$, where

$$u = u_n = \exp(hn^{-1} + xn^{-1/2}),\ d = d_n = \exp(hn^{-1} - yn^{-1/2}) \tag{2.11}$$

and

$$s_{k,n} = s_0 \xi_{1,n} \cdots \xi_{k,n}. \tag{2.12}$$

Note that the initial state of the model is determined by s_0 and the latest observed jump $\xi_0 = \exp(hn^{-1} + n^{-1/2}g(i_0))$. Since the chain is ergodic the influence of the initial value i_0 is asymptotically negligible.

In what follows we deal only with the call and put options defined by the pay-off function, respectively,

$$f(s) = (s - K)_+, \qquad f(s) = (K - s)_+. \tag{2.13}$$

The constant $K > 0$ is called the *strike price.*

Define

$$\psi(t, z) = \frac{x + y}{\sqrt{xy(1 - t)}}\varphi\left(\frac{\ln K - z + (1 - t)(xy/2 - \alpha)}{\sqrt{xy(1 - t)}}\right) \tag{2.14}$$

and

$$I(t) = \mathrm{E}\psi(t, z(t) + \ln s_0) = \frac{1}{\sqrt{t\sigma^2 + xy(1-t)}}\varphi\left(\frac{\ln(K/s_0) - ht + (1-t)(xy/2 - \alpha)}{\sqrt{t\sigma^2 + xy(1-t)}}\right). \tag{2.15}$$

Here, $\varphi(v)$ is the density function of the standard normal law.

Theorem 2.3 *Let the set $I = \{g(1), \ldots, g(M)\}$ is not lattice. If the conditions (2.9) and (2.10) are fulfilled then for any $i_0 > 0$ as $n \to \infty$*

$$\mathrm{E}_{i_0}\Delta_n = \frac{K}{2}(xy - \sigma_\pi^2)\int_0^1 I(t)\mathrm{d}t + o(1),$$

where

$$\sigma_\pi^2 = \sum_{i=1}^{M} g^2(i)\pi_i$$

and K is the strike price from (2.13).

Since g takes more than two values we have $\sigma_\pi^2 < xy$. So, the limit value in Theorem 2.3 is strictly positive. It should be emphasized that this limit value depends on x and y through xy. Furthermore, the upper rational price corresponding to x and y as $n \to \infty$ converges to

$$\bar{x}(0) \to c(xy) = s_0\Phi\left(\frac{\ln(s_0/K) + \alpha + xy/2}{\sqrt{xy}}\right) - Ke^{-\alpha}\Phi\left(\frac{\ln(s_0/K) + \alpha - xy/2}{\sqrt{xy}}\right).$$

As to the lower rational price given by the formula

$$\underline{x}_0 = \rho^{-n}(s_0\rho^n - K)_+$$

it converges to

$$c(0) = s_0\left(1 - \frac{K}{s_0 e^\alpha}\right)_+.$$

Below in Section 6 we show that the function $c(v)$ monotonically increases and, as expected, the Black-Scholes rational price $c(\sigma^2) \in (c(0), c(xy))$. It is worth reminding that if the random variables $\eta_1, \eta_2, \ldots$ are independent then $\sigma_\pi = \sigma$.

3 "Local" profit of investor

Here, we slightly modify the calculations presented in A. Nagaev and S. Nagaev (2003) and S. Nagaev (2003) (see Section 3 therein). Furthermore, we show that the required representation (3.24) for the "local" profit is the same for both payoff functions (2.13).

Let us convene to denote by c any positive constant whose concrete value is of no importance. Under such a convention we have e.g. $c+c=c,\ c^2=c$ etc. By $[\cdot,\cdot],((\cdot,\cdot])$ we denote a closed (closed from the right) interval and by θ any variable taking values in $[-1,1]$. By $[\cdot]$ and $\{\cdot\}$ we denote, respectively, the integer and fractional part of the embraced number.

Denote

$$p_n = \frac{\rho_n - d_n}{u_n - d_n},\ \ \lambda_{k,n} = \frac{\xi_{k,n} - d_n}{u_n - d_n}$$

and

$$a_{j,m} = u_n^j d_n^{m-j},\ \ b_{j,m} = C_m^j p_n^j (1-p_n)^{m-j}.$$

From (1.6) it follows that the discounted "local" profit of the investor takes the form

$$\Delta_{k,n} = \delta_{k,n}\rho_n^{n-k} = \sum_{j=0}^{n-k} b_{j,n-k}(\lambda_{k,n} f(s_{k-1,n}u_n a_{j,n-k}) + (1-\lambda_{k,n}) f(s_{k-1,n}d_n a_{j,n-k}) -$$

$$- f(s_{k-1,n}\xi_{k,n}a_{j,n-k})). \tag{3.16}$$

For time being we suppress the dependence of $\lambda_k,\ d,\ u,\ \xi_k$ and s_k on n.

Define

$$\bar{r}_m(z,Z) = \frac{\ln(Z/(zd^m))}{\ln(u/d)}.$$

The following lemma plays an important role.

Lemma 3.1 *If* $0 < x' \le \min(x,y) \le \max(x,y) \le x'' < \infty$ *then for* $d \le z \le u,\ m \le n$

$$\bar{r}_m(z,Z) = m \cdot \frac{y}{x+y} + n^{1/2}\left(\frac{Z}{x+y} - \frac{m+1}{n} \cdot \frac{h}{x+y}\right) - \frac{w}{x+y}$$

where $\ln z = hn^{-1} + wn^{-1/2}$.

Proof. From (2.11) it follows that

$$\ln\frac{u}{d} = (x+y)n^{-1/2}$$

and, therefore,

$$\frac{\ln z}{\ln(u/d)} = \frac{w}{x+y} + \frac{h}{x+y} \cdot n^{-1/2}.$$

In particular,

$$\frac{\ln d}{\ln(u/d)} = -\frac{y}{x+y} + \frac{h}{x+y} \cdot n^{-1/2}$$

and the lemma follows.

It is easily seen that $\bar{r}_m(d,Z)-\bar{r}_m(u,Z)=1$. Moreover,

$$\#\{j:\ \bar{r}_m(u,Z)<j\le \bar{r}_m(d,Z)\}=1. \tag{3.17}$$

Taking into account (2.10) and (2.11) we obtain

$$u-d=(x+y)n^{-1/2}+\frac{x^2-y^2}{2}n^{-1}+O(n^{-3/2})$$

while

$$\rho-d=yn^{-1/2}+(\alpha-h-y^2/2)n^{-1}+O(n^{-3/2}).$$

Therefore,

$$p_n=\frac{y}{x+y}+\frac{\alpha-h-xy/2}{x+y}n^{-1/2}+O(n^{-1}).$$

By Lemma 3.1

$$r_m(d,Z)-mp_n=n^{1/2}\left(\frac{\ln Z}{x+y}+\frac{m}{n}(\frac{xy}{2(x+y)}-\frac{\alpha}{x+y})\right)+O(1)$$

and, therefore,

$$\frac{r_m(d,Z)-mp_n}{\sqrt{mp_n(1-p_n)}}=(m/n)^{-1/2}(xy)^{-1/2}\left(\ln Z+(m/n)(\frac{xy}{2}-\alpha)\right)+O(m^{-1/2}). \tag{3.18}$$

3.1 The case of the call option

Assume that the payoff function is of the form $f(s)=(s-K)_+$. For the sake of brevity put

$$r_{n-k}(z)=\bar{r}_{n-k}(z,K/s_{k-1}).$$

Let j be such that $s_{k-1}da_{j,n-k}>K$. Then

$$\lambda_k f(s_{k-1}ua_{j,n-k})+(1-\lambda_k)f(s_{k-1}da_{j,n-k})-f(s_{k-1}\xi_k a_{j,n-k})=s_{k-1}(\lambda_k u+(1-\lambda_k)d-\xi_k)a_{j,n-k}=0.$$

If $s_{k-1}ua_{j,n-k}\le K$ then

$$0=f(s_{k-1}ua_{j,n-k})\ge f(s_{k-1}\xi_k a_{j,n-k})\ge f(s_{k-1}da_{j,n-k}).$$

It is worth reminding that $d\le\xi_{k-1}\le u$. Thus,

$$\Delta_{k,n}=\delta_{k,n}\rho_n^{n-k}=\sum_{r_{n-k}(u)<j\le r_{n-k}(d)} b_{j,n-k}(\lambda_k(s_{k-1}ua_{j,n-k}-K)_+ +$$

$$(1-\lambda_k)(s_{k-1}da_{j,n-k}-K)_+-(s_{k-1}\xi_k a_{j,n-k}-K)_+).$$

Further,

$$\Delta_{k,n} = \sum_{r_{n-k}(\xi_k)<j\le r_{n-k}(d)} b_{j,n-k}(\lambda_k(s_{k-1}ua_{j,n-k}-K)-(s_{k-1}\xi_k a_{j,n-k}-K))+$$

$$\lambda_k \sum_{r_{n-k}(u)<j\le r_{n-k}(\xi_k)} b_{j,n-k}(s_{k-1}ua_{j,n-k}-K) = \Delta'_{k,n}+\Delta''_{k,n}. \tag{3.19}$$

By definition of $r_{n-k}(z)$ we have

$$s_{k-1}za_{j,n-k} = s_{k-1}zd^{n-k}(u/d)^j = K(u/d)^{j-r_{n-k}(z)}.$$

Hence

$$s_{k-1}ua_{j,n-k} = K(u/d)^{j-r_{n-k}(u)} = K(u/d)^{j+1-r_{n-k}(d)}$$

and

$$s_{k-1}da_{j,n-k} = K(u/d)^{j-r_{n-k}(d)}.$$

Since $\lambda_k u-\xi_k = -d(1-\lambda_k)$ we conclude that

$$\Delta'_{k,n} = (1-\lambda_k)K \sum_{r_{n-k}(\xi_k)<j\le r_{n-k}(d)} b_{j,n-k}\left(1-(d/u)^{r_{n-k}(d)-j}\right) \tag{3.20}$$

while

$$\Delta''_{k,n} = \lambda_k K \sum_{r_{n-k}(u)<j\le r_{n-k}(\xi_k)} b_{j,n-k}\left((u/d)^{j+1-r_{n-k}(d)}-1\right). \tag{3.21}$$

In view of (2.11) and (3.17) we have uniformly in $k,\ \delta n\le k\le(1-\delta)n$,

$$1-(d/u)^{r_{n-k}(d)-j} = (x+y)n^{-1/2}(r_{n-k}(d)-j+O(n^{-1}))$$

and

$$(u/d)^{j+1-r_{n-k}(d)}-1 = (x+y)n^{-1/2}(j+1-r_{n-k}(d)+O(n^{-1})).$$

Here $\delta>0$ is arbitrarily small.

Taking into account (2.11) we conclude that

$$\Delta'_{k,n} = K(x-\eta_k+O(n^{-1/2}))n^{-1/2} \sum_{r_{n-k}(\xi_k)<j\le r_{n-k}(d)} b_{j,n-k}(r_{n-k}(d)-j+O(n^{-1}))$$

while

$$\Delta''_{k,n} = K(\eta_k+y+O(n^{-1/2}))n^{-1/2} \sum_{r_{n-k}(u)<j\le r_{n-k}(\xi_k)} b_{j,n-k}(j+1-r_{n-k}(d)+O(n^{-1})).$$

Both representations are valid uniformly in $k,\ \delta n\le k\le(1-\delta)n$.

By the uniform version of the Moivre-Laplace local limit theorem we obtain for $k,\ \delta n\le k\le(1-\delta)n$,

$$b_{j,n-k} = \frac{1}{\sqrt{(n-k)p_n(1-p_n)}}\varphi\left(\frac{j-(n-k)p_n}{\sqrt{(n-k)p_n(1-p_n)}}) + o(n^{-1/2}\right)$$

or, taking into account (3.18) and (2.14)

$$b_{j,n-k} = n^{-1/2}\psi(kn^{-1}, \ln s_{k-1}) + o(n^{-1/2}). \tag{3.22}$$

It is worth emphasizing that (3.22) holds uniformly in s_{k-1}.

Thus,

$$\Delta'_{k,n} = K(x-\eta_k)n^{-1}\psi(kn^{-1}, \ln s_{k-1}) \sum_{r_{n-k}(\xi_k)<j\le r_{n-k}(d)} (r_{n-k}(d)-j) + O(n^{-3/2})$$

while

$$\Delta''_{k,n} = K(\eta_k+y)n^{-1}\psi(kn^{-1}, \ln s_{k-1}) \sum_{r_{n-k}(u)<j\le r_{n-k}(\xi_k)} (j+1-r_{n-k}(d)) + O(n^{-3/2}).$$

Both representations are valid uniformly in k, $\delta n \le k \le (1-\delta)n$. In view of (3.17) the interval $(r_{n-k}(u), r_{n-k}(d)]$ contains exactly one integer $j^* = [r_{n-k}(d)]$. So,

$$\sum_{r_{n-k}(\xi_k)<j\le r_{n-k}(d)} (r_{n-k}(d)-j) = \begin{cases} \{r_{n-k}(d)\} & \text{if } r_{n-k}(\xi_k) < [r_{n-k}(d)] \\ 0 & \text{otherwise.} \end{cases}$$

Similarly,

$$\sum_{r_{n-k}(u)<j\le r_{n-k}(\xi_k)} (j+1-r_{n-k}(d)) = \begin{cases} 0 & \text{if } r_{n-k}(\xi_k) < [r_{n-k}(d)] \\ 1-\{r_{n-k}(d)\} & \text{otherwise.} \end{cases}$$

It is worth reminding that $\{r_{n-k}(d)\}$ denotes the fractional part of $r_{n-k}(d)$. Denote

$$\sigma_{k,n} = \begin{cases} (x-\eta_k)\{r_{n-k}(d)\} & \text{if } r_{n-k}(\xi_k) < [r_{n-k}(d)] \\ (\eta_k+y)(1-\{r_{n-k}(d)\}) & \text{otherwise.} \end{cases}$$

For the sake of brevity put

$$p = \frac{y}{x+y}, \;\; R = x+y.$$

Then the inequality $r_{n-k}(\xi_k) < [r_{n-k}(d)]$ can be rewritten as

$$\eta_k > R(\{r_{n-k}(d)\} - p).$$

Therefore,

$$\sigma_{k,n} = \begin{cases} (x-\eta_k)\{r_{n-k}(d)\} & \text{if } \eta_k > R(\{r_{n-k}(d)\}-p) \\ \\ (\eta_k+y)(1-\{r_{n-k}(d)\}) & \text{otherwise.} \end{cases} \tag{3.23}$$

Now, we may combine (3.19) and the latest estimates in the following way

$$\Delta_{k,n} = Kn^{-1}\psi(kn^{-1}, \ln s_{k-1})\sigma_{k,n} + O(n^{-3/2}). \tag{3.24}$$

So, we obtained the desired representation of the "local" profit in the case of the call option. It is of interest that its representation does not depend on the measure governing the price evolution.

3.2 The case of the put option

Assume that the payoff function is of the form $f(s) = (K-s)_+$. Let j be such that $s_{k-1}da_{j,n-k} \le K$. Then

$$\lambda_k f(s_{k-1}ua_{j,n-k}) + (1-\lambda_k)f(s_{k-1}da_{j,n-k}) - f(s_{k-1}\xi_k a_{j,n-k}) = s_{k-1}(\lambda_k u + (1-\lambda_k)d - \xi_k)a_{j,n-k} = 0.$$

If $s_{k-1}da_{j,n-k} \ge K$ then

$$f(s_{k-1}ua_{j,n-k}) = f(s_{k-1}\xi_k a_{j,n-k}) = f(s_{k-1}da_{j,n-k}).$$

Recall that $d \le \xi_{k-1} \le u$. Thus,

$$\Delta_{k,n} = \delta_{k,n}\rho_n^{n-k} = \sum_{r_{n-k}(u)<j\le r_{n-k}(d)} b_{j,n-k}(\lambda_k(K - s_{k-1}ua_{j,n-k})_+ +$$

$$(1-\lambda_k)(K - s_{k-1}da_{j,n-k})_+ - (K - s_{k-1}\xi_k a_{j,n-k})_+).$$

Further,

$$\Delta_{k,n} = (1-\lambda_k)\sum_{r_{n-k}(\xi_k)<j\le r_{n-k}(d)} b_{j,n-k}(K - s_{k-1}da_{j,n-k}) +$$

$$\sum_{r_{n-k}(u)<j\le r_{n-k}(\xi_k)} b_{j,n-k}((1-\lambda_k)(K - s_{k-1}da_{j,n-k}) - (K - s_{k-1}\xi_k a_{j,n-k})) = \Delta'_{k,n} + \Delta''_{k,n}. \tag{3.25}$$

Since $(1-\lambda_k)d - \xi_k = -u\lambda_k$ we arrive at (3.20) and (3.21).

Now it remains to repeat the calculations leading to (3.24). Thus, the "local" profits of the put and call options admit the same representation.

4 Proof of Theorem 2.3

Represent the total profit Δ_n as

$$\Delta_n = \sum_{1\le k<\delta n} \Delta_{k,n} + \sum_{\delta n\le k\le(1-\delta)n} \Delta_{k,n} + \sum_{(1-\delta)n\le k\le n} \Delta_{k,n} = \Delta_n' + \Delta_n'' + \Delta_n''' \tag{4.26}$$

and estimate the expectations $\mathrm{E}\Delta_n'$, $\mathrm{E}\Delta_n''$ and $\mathrm{E}\Delta_n'''$ one after another.

According to (3.24) we have

$$\mathrm{E}_{i_0}\Delta_n'' = Kn^{-1} \sum_{\delta n\le k\le(1-\delta)n} \mathrm{E}\psi(kn^{-1}, \ln s_{k-1,n})\sigma_{k,n} + c\theta n^{-1/2}.$$

Consider

$$A(u,v) = (x-v)u\chi(u,v) + (v+y)(1-u)(1-\chi(u,v)),\ (u,v)\in[0,1]\times[-y,x], \tag{4.27}$$

where

$$\chi(u,v) = \begin{cases} 1 & \text{if} \quad R(u-p) < v \le x,\ 0\le u\le 1 \\ 0 & \text{if} \quad -y < v \le R(u-p),\ 0\le u\le 1 \end{cases}$$

In view of (3.23) we have

$$\sigma_{k,n} = A(\{r_{n-k}(d)\}, \eta_k).$$

It is evident that $\chi(u,v)$ admits a monotone $\varepsilon-$approximation by means of $\chi_+(u,v)$ and $\chi_-(u,v)$ where

$$\chi_+(u,v) = \begin{cases} \frac{v-R(u-p)}{\varepsilon} + 1 & \text{if } R(u-p)-\varepsilon \le v \le R(u-p),\ 0\le u\le 1 \\ 0 & \text{if } -y \le v \le R(u-p)-\varepsilon,\ 0\le u\le 1 \\ 1 & \text{if } R(u-p) \le v \le x,\ 0\le u\le 1 \end{cases}$$

and

$$\chi_-(u,v) = \begin{cases} \frac{v-R(u-p)}{\varepsilon} & \text{if } R(u-p) \le v \le R(u-p)+\varepsilon,\ 0\le u\le 1 \\ 0 & \text{if } -y \le v \le R(u-p),\ 0\le u\le 1 \\ 1 & \text{if } R(u-p)+\varepsilon \le v \le x,\ 0\le u\le 1. \end{cases}$$

Obviously, $\chi_{\pm}(u,v)$ are continuous in $[0,1]\times[-y,x]$ and

$$\chi_-(u,v)\le\chi(u,v)\le\chi_+(u,v).$$

Furthermore,

$$0\le\int\limits_{[0,1]\times[-y,x]}(\chi_+(u,v)-\chi_-(u,v))\mathrm{d}u\mathrm{d}F(v)\le\int\limits_{U_\varepsilon}\mathrm{d}u\mathrm{d}F(v)\le(2\varepsilon/R) \qquad (4.28)$$

where

$$U_\varepsilon=((u,v):\ u\in(0,1),\ -y<v<x,\ |v-R(u-p)|\le\varepsilon).$$

Therefore

$$\mathrm{E}_{i_0}\left(\psi(kn^{-1},\tfrac{\eta_1+\cdots+\eta_{k-1}}{\sqrt{n}}+h\tfrac{k-1}{n}+\ln s_0)A_-(\{r_{n-k}(d)\},\eta_k)|\eta_{k-1}=g(i)\right)\le$$

$$\mathrm{E}_{i_0}\left(\psi(kn^{-1},\tfrac{\eta_1+\cdots+\eta_{k-1}}{\sqrt{n}}+h\tfrac{k-1}{n}+\ln s_0)\sigma_{k,n}|\eta_{k-1}=g(i)\right)=$$

$$\mathrm{E}_{i_0}\left(\psi(kn^{-1},\tfrac{\eta_1+\cdots+\eta_{k-1}}{\sqrt{n}}+h\tfrac{k-1}{n}+\ln s_0)A(\{r_{n-k}(d)\},\eta_k)|\eta_{k-1}=g(i)\right)\le$$

$$\mathrm{E}_{i_0}\left(\psi(kn^{-1},\tfrac{\eta_1+\cdots+\eta_{k-1}}{\sqrt{n}}+h\tfrac{k-1}{n}+\ln s_0)A_+(\{r_{n-k}(d)\},\eta_k)|\eta_{k-1}=g(i)\right)$$

where

$$A_{\pm}(u,v)=(x-y)u\chi_{\pm}(u,v)+(y+x)(1-u)(1-\chi_{\mp}(u,v)).$$

Obviously, the family $\psi(t,z),\ \delta\le t\le1-\delta$, is contained in the class $\mathcal{F}$ defined in Corollary 5.2.

By the corollary

$$\mathrm{E}_{i_0}\left(\psi(kn^{-1},\tfrac{\eta_1+\cdots+\eta_{k-1}}{\sqrt{n}}+h\tfrac{k-1}{n}+\ln s_0)A_{\pm}(\{r_{n-k}(d)\},\eta_k)|\eta_{k-1}=g(i)\right)=$$

$$\mathrm{E}\psi(kn^{-1},\sigma\nu\sqrt{kn^{-1}}+hkn^{-1}+\ln s_0)\int\limits_{[0,1]\times[-y,x]}A_{\pm}(u,v)\mathrm{d}u\mathrm{d}\mathrm{P}(\eta_k<v|\eta_{k-1}=g(i))+o(1)$$

uniformly in $k,\ \delta\le kn^{-1}\le1-\delta$. Here ν has the standard $(0,1)-$normal distribution and F is the distribution function of η.

In view of (4.28)

$$\int\limits_{[0,1]\times[-y,x]}A_{\pm}(u,v)\mathrm{d}u\mathrm{d}\mathrm{P}(\eta_k<v|\eta_{k-1}=g(i))=\int\limits_{[0,1]\times[-y,x]}A(u,v)\mathrm{d}u\mathrm{d}\mathrm{P}(\eta_k<v|\eta_{k-1}=g(i))+2\theta\varepsilon.$$

It is easily verified that

$$\int A(z,w)\mathrm{d}\mathrm{P}(\eta_k\le w|\eta_{k-1}=g(i))=\frac{1}{2(x+y)}(xy+(x-y)a_i-b_i^2)$$

where

$$a_i = \sum_{l=1}^{M} g(l)p_{il}, \; b_i^2 = \sum_{l=1}^{M} g^2(l)p_{il}.$$

Since ε is arbitrary we obtain

$$\mathrm{E}_{i_0}\sigma_{k,n} =$$

$$\tfrac{1}{2(x+y)}(xy + (x-y)\textstyle\sum_{i=1}^{M} a_i \mathrm{P}_{i_0}(\eta_{k-1} = g(i)) - \textstyle\sum_{i=1}^{M} b_i^2 \mathrm{P}_{i_0}(\eta_{k-1} = g(i))) + o(1).$$

Since the chain is ergodic we have as $k \to \infty$

$$\mathrm{P}_{i_0}(\eta_{k-1} = g(i)) = \pi_i + o(1)$$

whence

$$\sum_{i=1}^{M} a_i \mathrm{P}_{i_0}(\eta_{k-1} = g(i)) = \sum_{i=1}^{M} a_i\pi_i + o(1) = \sum_{i=1}^{M} \pi_i \sum_{l=1}^{M} g(l)p_{il} + o(1) = \sum_{l=1}^{M} g(l)\pi_l + o(1) = o(1).$$

Similarly,

$$\sum_{i=1}^{M} b_i^2 \mathrm{P}_{i_0}(\eta_{k-1} = g(i))) = \sum_{l=1}^{M} g^2(l)\pi_l + o(1) = \sigma_\pi^2 + o(1).$$

Thus,

$$\mathrm{E}_{i_0}\Delta_n'' = \frac{K}{2(x+y)}(xy - \sigma_\pi^2)n^{-1} \sum_{\delta n \le k \le (1-\delta)n} \mathrm{E}\psi(kn^{-1}, \sigma\nu\sqrt{kn^{-1}} + hkn^{-1} + \ln s_0) + o(1).$$

Obviously,

$$I(t) = \mathrm{E}\psi(t, \sigma\nu\sqrt{t} + ht + \ln s_0) = \int \psi(t, \sigma v\sqrt{t} + ht + \ln s_0)\varphi(v)\mathrm{d}v$$

or after simple calculations

$$I(t) = \frac{x+y}{\sqrt{t\sigma^2 + xy(1-t)}}\varphi\left(\frac{\ln(K/s_0) - ht + (1-t)(xy/2 - \alpha)}{\sqrt{t\sigma^2 + xy(1-t)}}\right)$$

whence we deduce

$$\mathrm{E}_{i_0}\Delta_n'' = \frac{K}{2}(xy - \sigma_\pi^2)\int_{\delta}^{1-\delta} I(t)\mathrm{d}t + o(1). \tag{4.29}$$

Now we are going to estimate $\mathrm{E}\Delta_n'''$. For the extreme "local" profit $\Delta_{n,n}$ we obtain

$$\Delta_{n,n} = \delta_{n,n} = (s_{n-1,n}d_n - K)_+ \frac{u_n - \xi_n}{u_n - d_n} + (s_{n-1,n}u_n - K)_+ \frac{\xi_n - d_n}{u_n - d_n} - (s_{n-1,n}\xi_{n.n} - K)_+$$

whence

$$\Delta_{n,n} = \begin{cases} 0 & \text{if} \quad s_{n-1,n}u_n \le K \text{ or } s_{n-1,n}d_n > K \\ \theta(s_{n-1,n}u_n - K) & \text{if} \quad K/u_n < s_{n-1,n} \le K/d_n. \end{cases}$$

Therefore,

$$\mathrm{E}_{i_0}\Delta_{n,n} \le K(u_n/d_n - 1) \le cn^{-1/2}.$$

For $m = n - k \ge 1$ in view of (3.19) – (3.21)

$$\Delta_{n-m,n} \le c \max_j b_{j,m} \left((u_n/d_n)^2 - 1\right)$$

or taking into account (2.11) and (2.2)

$$\Delta_{n-m,n} \le cm^{-1/2}n^{-1/2}.$$

Thus, for all sufficiently large n

$$\mathrm{E}_{i_0}\Delta_n''' \le c\delta^{1/2}. \tag{4.30}$$

Similarly,

$$\mathrm{E}_{i_0}\Delta_n' \le c\delta. \tag{4.31}$$

Since δ is arbitrary in view of (4.26), (4.29), (4.30) and (4.31) the theorem follows.

5 Auxiliary statements

The following statement is a Markov chain analogue of Lemma 7.1 in A. Nagaev and S. Nagaev (2003).

Lemma 5.1 *Under the conditions of Theorem 2.3 the random variables* $\eta_{k-1},\ \{r_{n-k}(d)\}$ *and* $s_{k-1,n}$ *are asymptotically independent as* $n \to \infty$ *in the sense that for any* $\delta > 0$ *as* $n \to \infty$

$$\sup_{0<\delta\le kn^{-1}\le 1-\delta\ v,} \ \sup_{z\in[0,1]} |\mathrm{P}_{i_0}(\eta_{k-1} = g(i),\ \{r_{n-k}(d)\} \le z,\ \ln s_{k-1,n} \le v) -$$

$$\pi_i z \mathrm{P}(hkn^{-1} + \sigma w(kn^{-1}) < v)| = o(1).$$

Proof. From Proposition 2.2 it follows that as $n \to \infty$

$$\mathrm{P}_{i_0}(v' \le \eta_1 + \cdots + \eta_n - x < v'') = \frac{v'' - v'}{\sigma\sqrt{n}} \phi\left(\frac{x}{\sigma\sqrt{n}}\right) + o(n^{-1/2}) \tag{5.32}$$

uniformly in $v', v'',\ 0 < c_1 \le v'' - v' \le c_2 < \infty$. As in A. Nagaev and S. Nagaev (2003)(see Lemma 7.1 therein) we put

$$\zeta_n = \eta_1, + \cdots + \eta_n,\ \tau_n(a) = \{\lambda\zeta_n + a\}$$

where λ is constant. In view of (5.32) the proof of Lemma 7.1 in A. Nagaev and S. Nagaev (2003) remains valid for the considered Markov chain. So, for any fixed $u', u'',\ 0 < u' < u'' < 1$ and $z', z'',\ -\infty < z' < z'' < \infty$ as $n \to \infty$

$$[\mathrm{P}_{i_0}(u' \le \tau_n(a) < u'',\ z' \le n^{-1/2}\zeta_n < z'') - (u'' - u')\left(\Phi(z''/\sigma) - \Phi(z'/\sigma)\right) = o(1) \tag{5.33}$$

uniformly in $a \in I\!R$. From Lemma 3.1 it follows that

$$r_{n-k}(d) = \lambda\zeta_{k-1} + a$$

where

$$\lambda = -\frac{1}{x+y},\ a = (n-k+1)p - n^{1/2} \cdot \frac{h}{x+y} + \frac{\ln(K/s_0)}{x+y}.$$

Further,

$$\mathrm{P}_{i_0}(\eta_{k-1} = g(i),\ \{r_{n-k}(d)\} \le z,\ \ln s_{k-1,n} \le v) = \mathrm{P}_{i_0}(\eta_{k-1} = g(i)) \times$$

$$\mathrm{P}_{i_0}(\{\lambda\zeta_{k-2} + a + \lambda g(i)\} \le v,\ \tfrac{\eta_1 + \cdots + \eta_{k-2}}{\sqrt{n}} + \tfrac{(k-1)h}{n} + \tfrac{g(i)}{\sqrt{n}} < v|\ \eta_{k-1} = g(i)).$$

From (5.33) it follow that

$$\mathrm{P}_{i_0}(\{\lambda\zeta_{k-2} + a + \lambda g(i)\} \le v,\ \tfrac{\eta_1 + \cdots + \eta_{k-2}}{\sqrt{n}} + \tfrac{(k-1)h}{n} + \tfrac{g(i)}{\sqrt{n}} < v|\ \eta_{k-1} = g(i)) =$$

$$z\mathrm{P}(khn^{-1} + \sqrt{k\sigma^2 n^{-1}}\nu < v) + o(1)$$

uniformly in a and $k,\ 0 < \delta \le kn^{-1} \le 1 - \delta$. As above, the random variable ν has the standard normal distribution. Since the Markov chain is ergodic we have as $k \to \infty$

$$\mathrm{P}_{i_0}(\eta_{k-1} = g(i)) = \pi_i + o(1)$$

and the lemma follows.

Lemma 5.1 has the following evident corollary (cf. Corollary 7.2 in A. Nagaev and S. Nagaev (2003)).

Corollary 5.2 *Let $\mathcal{F}$ be the class of equicontinuous functions defined on* $(-\infty, \infty)$ *such that*

$$\lim_{t\to\infty} \sup_{f\in\mathcal{F}} \int\limits_{|u|>t} |f(u)|\mathrm{d}u = 0.$$

Let, further, $\chi(u,v)$ be a bounded continuous function defined on $[0,1]\times I\!R^1$. *Under the conditions of Lemma 5.1 for any fixed λ as $n \to \infty$*

$$\lim_{n\to\infty} \sup_{f\in\mathcal{F}} |\mathrm{E}_{i_0}\left(f(n^{-1/2}\zeta_{k-1})\chi(\{r_{n-k}(d)\},\eta_k)|\eta_{k-1} = g(i)\right) -$$

$$\int f(\sigma z)\varphi(z)\mathrm{d}z \int\limits_{[0,1]\times I\!R^1} \chi(u,v)\mathrm{d}u\mathrm{d}\mathrm{F}_i(v)| = 0$$

uniformly k, $0 < \delta \le kn^{-1} \le 1-\delta$. *Here*

$$F_i(v) = \mathrm{P}(\eta_n < v|\eta_{n-1} = g(i)).$$

6 Concluding remarks

First, note that the representation (3.24) of the "local" profit assumes no specification of the measure that governs the evolution of the stock price. In other words, it is the same for any joint distribution of $\eta_1, \eta_2, \dots, \eta_n$. The further analysis of (3.24) is based on the functional central limit theorem for the successive sums

$$\zeta_n = \eta_1 + \eta_2 + \cdots + \eta_k,\ k = 1, 2, \dots,$$

and the local limit theorem given by Proposition 2.2. It is the local limit theorem that allows successful analysis of the chaotic term $\sigma_{n,k}$ in (3.24). If the joint distribution of $\eta_1, \eta_2, \dots, \eta_n$ admits those fundamental limit theorems then the suggested method works well. So, the basic problem now is to verify whether a given discrete time process $\eta_1, \eta_2, \dots, \eta_n, \dots$ possesses the required property. In particular, it is worth trying to analyze such popular stochastic models as, say, ARIMA or GARCH.

Further, consider the limit value of the upper rational price

$$c(xy) = s_0\Phi\left(\frac{\ln(s_0/K) + \alpha + xy/2}{\sqrt{xy}}\right) - Ke^{-\alpha}\Phi\left(\frac{\ln(s_0/K) + \alpha - xy/2}{\sqrt{xy}}\right).$$

For the sake of brevity put $Z = K/(s_0e^{-\alpha}), xy = v^2$. Then

$$c(xy) = c(v^2) = s_0\left(\Phi\left(\frac{-\ln Z + v^2/2}{v}\right) - Z\Phi\left(\frac{-\ln Z - v^2/2}{v}\right)\right).$$

After a simple algebra we obtain

$$\mathrm{sign}(c'(v^2)) = \mathrm{sign}((Z-1)\ln Z + (1+Z)v^2/1).$$

It is easily seen that for all $Z > 0$ we have $c'(v^2) > 0$. Thus $c(xy)$ monotonically increases as xy grows. If g takes more than two values then $xy > \sigma_\pi$. This implies that

$$c(0) < c(\sigma^2) < c(xy). \tag{6.34}$$

Note that $c(\sigma^2)$ correspond to the Black-Scholes rational price.

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